Economic Principles for Decision Making

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Greening Energy Market and Finance

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Greening Energy Market and Finance
Summer School
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Road map

Outline

The Importance of Decision Making from a Personal Perspective

- The importance of decision making from a personal perspective:
  - As you go up in the ladder of your organization, you will do less and less "things," but you mill make more and more decisions

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  - Are you waiting to be motivated by your boss?

The Importance of Decision Making for Leaders

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Good leaders need to be good decision makers.

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  - Taking a route means you *will not* take another
  - This leads to opportunity costs

Opportunity costs

- Opportunity costs is the forgone benefit that would have been derived by an option not chosen.
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- Taking into account opportunity costs is just one of the economic principles for decision making
- I will not talk about how to communicate or *make propaganda* of the decisions, but only about how to make good decisions.
  - Of course, communicating your decisions is *extremely important!* — it is just not our topic.

The Importance of Decision Making for Organizations and Society

- In sum: if you want to be a good citizen, a good professional, etc...
- Then, please *decide* to become a good decision maker
- This course will try to teach a few basic principles about economic decision making
  - Contrast with behavioral economics
  - Decision making under certainty
  - Decision making under uncertainty
  - Applications to Financial decisions

Outline

Normative vs. descriptive

- It is important to understand the distinction between the normative and positive approaches to decision making:
  - Normative (prescriptive): it aims to establish how decisions *should* be made
  - Positive (descriptive): it studies how people actually decide
- Psychologists and behavioral economists usually follow the positive/descriptive approach
- You should take this difference into account when reading about decision making
- Both approaches are **useful**— for **different purposes**.
  - Normative (prescriptive): to learn how to make decisions
  - Positive (descriptive): to prevent pitfalls and mistakes that are common in how people make decisions

  Note: if you are dealing with a population (citizens, customers, etc.) you can also use this knowledge to nudge them. See Thaler and Sunstein (2009).

Daniel Kahneman's *Thinking, Fast and Slow*

Take for instance, Daniel Kahneman's bestseller: *Thinking, Fast and Slow*.

- Kahneman (2011) shows problems in decisions (and perceptions)

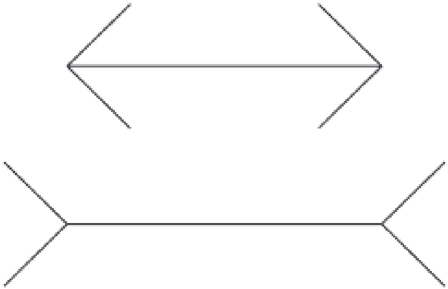

Figure: This is the famous Müller-Lyer illusion. The horizontal lines are in fact identical in length.

Another example: decision to pay for milk

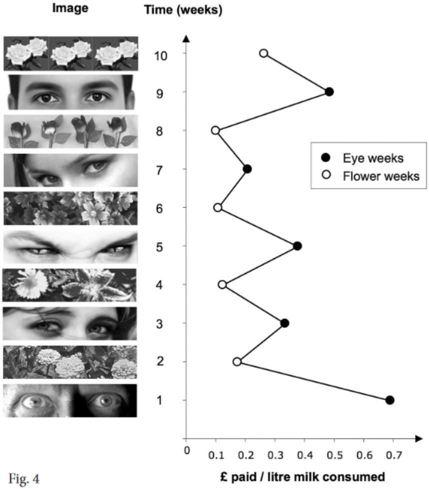

Fig. 4

Figure: On average, the users of an office kitchen contributed almost three times as much in "eyes weeks" as they did in "flowers weeks."

What we learn from behavioral economics?

- People have reference points and see outcomes with respect to those "reference points"
- Gains are valued less than losses — there is loss aversion
- People are "risk averse" in gains
- People are "risk seeking" in losses
- People distort the perception of probabilities

Knowing this, you can try to avoid those "mistakes"

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- Are those really mistakes?

Lindley on normative vs. descriptive

- "Mistakes" are relative to a "correct" way of making decisions
- Of course, this has to be a "normative" theory
- "Normative" theories exist – but they are not "perfect"

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"Psychologists are much concerned with the manner of this appreciation, and experiments have been performed which show that individuals do not reason about uncertainty in the way described in these volumes. The experiments provide a descriptive view of man's attitudes: de Finetti's approach is normative. To spend too much time on description is unwise when a normative approach exists, for it is like asking people's opinion of $2 + 2$, obtaining an average of 4.31 and announcing this to be the sum. It would be better to teach them arithmetic." Lindley (1974, p. vii)

Outline

Five Steps to Decision Making

1. Define the problem and the objective

   What is the problem the manager faces? Who is the decision maker? What is the decision setting or context, and how does it influence managerial objectives or options? What is the problem the manager faces? Who is the decision maker? What is the decision setting or context, and how does it influence managerial objectives or options?

Five Steps to Decision Making

1. Define the problem and the objective
2. Explore the alternatives

   What are the alternative courses of action? What are the variables under the decision maker's control? What constraints limit the choice of options?

Five Steps to Decision Making

1. Define the problem and the objective
2. Explore the alternatives
3. Predict the consequences

What are the consequences of each alternative action? Should conditions change, how would this affect outcomes? If outcomes are uncertain, what is the likelihood of each? Can better information be acquired to predict outcomes?

Five Steps to Decision Making

1. Define the problem and the objective
2. Explore the alternatives
3. Predict the consequences
4. Compare the consequences

How the consequences can be ranked/compared? What is the best course of action?

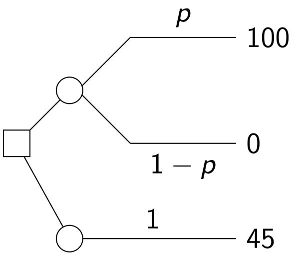
Five Steps to Decision Making

1. Define the problem and the objective
2. Explore the alternatives
3. Predict the consequences
4. Compare the consequences
5. Perform Sensitivity analysis

   What features of the problem determine the optimal choice of action? How does the optimal decision change if conditions in the problem are altered? Is the choice sensitive to key economic variables about which the decision maker is uncertain?

Decision Trees

- It is useful to visualize your decision in decision trees



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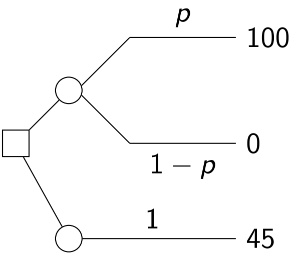
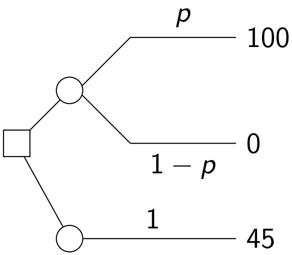


Decision Trees

- It is useful to visualize your decision in decision trees
- Decision trees are draw from the left to the right, and have only splitting nodes but no sink nodes (converging paths)
- Use squares (or rectangles) for node decisions and circles for chance events



Outline

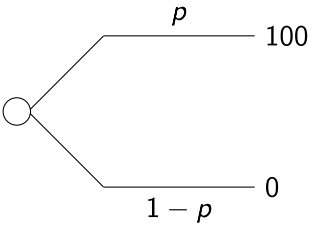
Let's go back to 1600s...

- Chevalier de Mère was a nobleman who gambled frequently
- He bet on a roll of a die that at least one 6 would appear during a total of four rolls
- From past experience, he knew that he was more successful than not in this bet
- He bet he would get a double 6 on 24 rolls of two dice
- Soon, he realized that this bet was not as profitable
- He asked his friend Blaise Pascal why
- Pascal developed a correspondence with Pierre de Fermat, and they are both credited with the founding of **probability theory**

The key idea: a bet should be evaluated by its **expected value**.

Calculating the Expected value

- Consider a bet that pays \$100 if some **event** happens, and nothing otherwise. It is represented below:



- What is its Expected Value?

$$E[X] = p \cdot 100 + (1-p) \cdot 0 = 100p.$$

Calculating the Expected value

- Suppose that the event is the first bet considered by de Mere, that is, to obtain a 6 if the dice is played four times. The probability of not getting 6 in four rolls is:

$$\left(\frac{5}{6}\right)^4 \approx 0.48 < \frac{1}{2}.$$

- Thus, the probability of getting at least a 6 is:

$$p = 1 - \left(\frac{5}{6}\right)^4 \approx 0.52 > \frac{1}{2}.$$

- The Expected Value of playing the bet of $100 in this event is therefore

$$E[X] = p \cdot 100 + (1 - p) \cdot 0 \approx 52 > 50.$$

Calculating the Expected value

- Now, assume the event is the second bet considered by de Mere, that is, to obtain a double 6 if the dice is played 24 times. The probability of not getting a double six in 24 rolls is:

$$\left(\frac{35}{36}\right)^{24} \approx 0.509 > \frac{1}{2}.$$

- Thus, the probability of winning the bet is:

$$p = 1 - \left(\frac{5}{6}\right)^{4} \approx 0.491 < \frac{1}{2}.$$

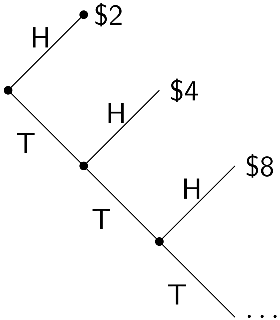
- The Expected Value of playing the bet of \$100 in this event is therefore

$$E[X] = p \cdot 100 + (1-p) \cdot 0 \approx 49.1 < 50.$$

So, is calculating the expectation the answer?

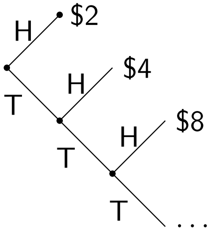
In 1713, Nicolas Bernoulli proposed the following bet:

- You toss a coin. In case of H, you receive \$2. In case of T, you toss it again.
- If H, you receive \$4. If T, you toss it again.
- In each play, the value is doubled.



A paradox!

- How much is this bet worth?
- How much would you pay to play it?



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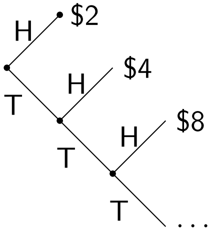


$$
\begin{aligned}
E[X] &= \frac{1}{2}2 + \frac{1}{4}4 + \frac{1}{8}8 + \frac{1}{16}16 + \ldots \\
&= 1 + 1 + 1 + 1 + \ldots \\
&= \infty!
\end{aligned}
$$

The birth of expected utility

- In 1738, Daniel Bernoulli (cousin of Nicolas Bernoulli), proposed a solution to the paradox:

  > *The determination of the value of an item must not be based on the price, but rather on the utility it yields... There is no doubt that a gain of one thousand ducats is more significant to the pauper than to a rich man though both gain the same amount.*

- Daniel Bernoulli lived in St. Petersburg when he published his solution—and this is how the paradox acquired its name

Expected Utility

- Bernoulli suggested using $U(x) = \ln(x)$.
- But other functions can be used, depending on the bet considered. A frequent choice is a power utility function: $U(x) = x^{\alpha}$, for $\alpha \in (0, 1)$

Example

Suppose that $U(x) = x^{0.5} = \sqrt{x}$ and let X be the random variable that has value 25 with probability 1/3 and 16 with probability 2/3. Then:

$$\begin{aligned} \mathrm{E}[U(X)] &= \frac{1}{3} \cdot \sqrt{25} + \frac{2}{3} \cdot \sqrt{16} \\ &= \frac{1}{3} \cdot 5 + \frac{2}{3} \cdot 4 \\ &= \frac{13}{3} = 4\frac{1}{3}. \end{aligned}$$

Certainty equivalent

Given $U(x)$ and a lottery X, the corresponding **certainty equivalent** is the value of the risk free lottery that makes the DM indifferent:

$$CE(X) = U^{-1}\left(\mathrm{E}[U(X)]\right).$$

Indeed, if $Y = CE(X)$ for sure, Y the same expected utility as X:

$$\mathrm{E}[U(Y)] = 1 \cdot U(CE(x)) = U\left[U^{-1}\left(\mathrm{E}[U(X)]\right)\right] = \mathrm{E}[U(X)].$$

Example (Continued)

Assuming the setting of the previous example, with $\mathrm{E}[U(x)] = \frac{13}{3}$, we have:

$$CE(X) = U^{-1}\left(\mathrm{E}[U(X)]\right) = \left(\mathrm{E}[U(X)]\right)^2 = \left(\frac{13}{3}\right)^2 = \frac{169}{9}.$$

Risk aversion

Definition

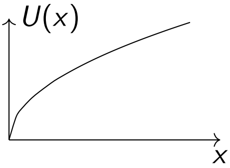
A DM is *risk averse* (or has *risk aversion*) if for all X,

$$CE(X) \leqslant \mathrm{E}[X].$$

- This means that the DM prefers a certain value slightly below the expectation of the lottery.

Theorem

The DM is risk averse if and only if $U(x)$ is concave.



This is how it looks like a concave utility function

Risk seeking

Definition

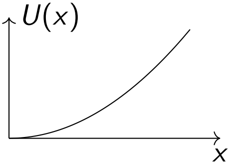
A DM is *risk seeking* (or risk loving) if for all X,

$$CE(X) \geqslant \mathrm{E}[X].$$

This means that the DM prefers the lottery to a certain value equals to the expectation of the lottery.

Theorem

The DM is risk seeking if and only if $U(x)$ is convex.



This is how it looks like a convex utility function

Risk neutrality

Definition

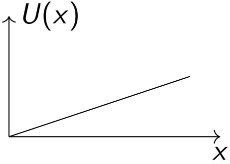
A DM is *risk neutral* (or has *risk neutrality*) if for all X,

$$CE(X) = \mathrm{E}[X].$$

In this case, the DM is indifferent between the lottery and its expectation.

Theorem

The DM is risk neutral if and only if $U(x)$ is affine.



The graph of an affine function is a line.

Risk attitude – summary

- A risk averse DM prefers $E[X]$ for sure to any lottery X.
- A risk seeking DM prefers the lottery X to $E[X]$ for sure.
- A risk neutral DM is indifferent between X and $E[X]$.

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- A risk seeking DM prefers the lottery X to E[X] for sure.
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Example

Let X be the lottery that pays 4 with probability $1/2$ and 0 with probability $1/2$. Then, $\mathrm{E}[X] = 2$. Let $U_1(x) = \sqrt{x}$, $U_2(x) = x^2$ and $U_3(x) = x$. Then:

$$\mathrm{E}[U_1(X)] = \frac{1}{2} \cdot \sqrt{4} = 1 < \sqrt{2} = U_1(2) \qquad \Rightarrow DM_1 \text{ is risk averse;}$$

$$\mathrm{E}[U_2(X)] = \frac{1}{2} \cdot 4^2 = 8 > 4 = 2^2 = U_2(2) \qquad \Rightarrow DM_2 \text{ is risk seeking;}$$

$$\mathrm{E}[U_3(X)] = \frac{1}{2} \cdot 4 = 2 = U_3(2) \qquad \Rightarrow DM_3 \text{ is risk neutral;}$$

Mean and Variance

In the analysis of the problem of portfolio selection, Markowitz (1952) was the first to consider **mean** and **variance** to select portfolios with different assets.

- **Mean**: it is just the Expected Value (expectation) that we have been considering:

$$\text{If } X \text{ is discrete: } \mathrm{E}[X] = \sum_{i=1}^{N} p_i x_i;$$

$$\text{If } X \text{ has a p.d.f. } f_X : \mathrm{E}[X] = \int \alpha f_X(\alpha) d\alpha.$$

- **Variance** is defined by

$$\mathrm{Var}[X] \equiv \mathrm{E}\left[(X - \mathrm{E}[X])^2\right] \geqslant 0.$$

Mean and Variance

- The Expectation has nice linearity properties, that is,

$$\mathrm{E}\left[\alpha X+\beta Y\right]=\alpha \mathrm{E}\left[X\right]+\beta \mathrm{E}\left[Y\right],$$

  for any random variables X and Y and real numbers $\alpha, \beta \in \mathbb{R}$.
- Using this property, we can simplify the expression of the variance

$$\begin{aligned}
\mathrm{Var}[X] &\equiv \mathrm{E}\left[(X-\mathrm{E}[X])^2\right] \\
&= \mathrm{E}\left[X^2-2X\mathrm{E}[X]+(\mathrm{E}[X])^2\right] \\
&= \mathrm{E}\left[X^2\right]-2\mathrm{E}\left[X\mathrm{E}[X]\right]+\mathrm{E}\left[(\mathrm{E}[X])^2\right] \\
&= \mathrm{E}\left[X^2\right]-2\mathrm{E}\left[X\right]\mathrm{E}[X]+(\mathrm{E}[X])^2 \\
&= \mathrm{E}\left[X^2\right]-(\mathrm{E}[X])^2 .
\end{aligned}$$

Mean and Variance

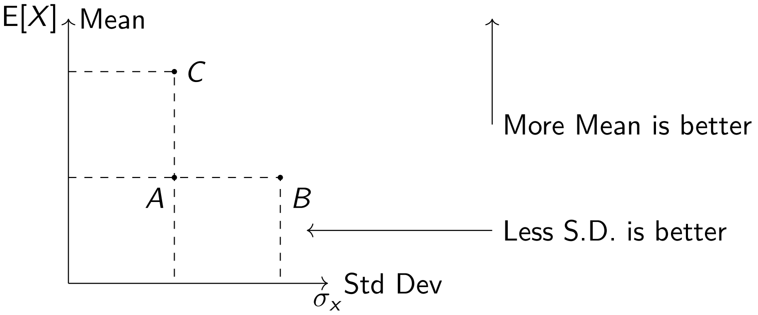
- The Variance indicates how risky an asset is. Indeed, if $X = \mathrm{E}[X]$ is risk free, we have

$$\mathrm{Var}[X] = \mathrm{E}\left[X^2\right] - (\mathrm{E}[X])^2 = X^2 - X^2 = 0.$$

  (Recall that the variance is always non-negative. Thus, risk free assets have minimal variance.)
- Therefore, it is natural to think that an investor likes the mean and dislikes the variance
- With this idea, Markowitz represented assets in a graph mean vs. variance and reasoned how to compare assets with different pair (mean, variance)

Obs.: Sometimes it is convenient to use the standard deviation $\sigma_X \equiv \sqrt{\mathrm{Var}[X]}$ as a measure of the risk of the asset X, instead of just its variance $\mathrm{Var}[X]$.

Preference between Assets



- A is preferable to B because it has lower standard deviation (it is less risky) and has the same mean;
- C is preferable to A because it has higher mean and the same standard deviation;
- B is preferable to C are incomparable using only Markowitz's criteria.

Mean and Variance vs. Expected Utility

- Notice that Markowitz's criteria is a methodology to compare lotteries/prospects or assets;
- But we have said that we could use Expected Utility just for that!
- Can we relate these two approaches?

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- Sure! Here, I will do this exercise using only the quadratic utility function $U(x) = ax - bx^2$, for $a, b > 0$ and X with support contained in $\left[0, \frac{a}{2b}\right]$.
- Let $\mu = \mathrm{E}[X]$ and $\sigma^2 = \mathrm{Var}[X] = \mathrm{E}[X^2] - (\mathrm{E}[X])^2$. We have:

$$\begin{aligned} \mathrm{E}[U(X)] &= \mathrm{E}[aX - bX^2] = a\mathrm{E}[X] - b\mathrm{E}\left[X^2\right] \\ &= a\mu - b\left(\sigma^2 + \mu^2\right). \end{aligned}$$

- We can obtain the set of points with the same expected utility (indifference curve).

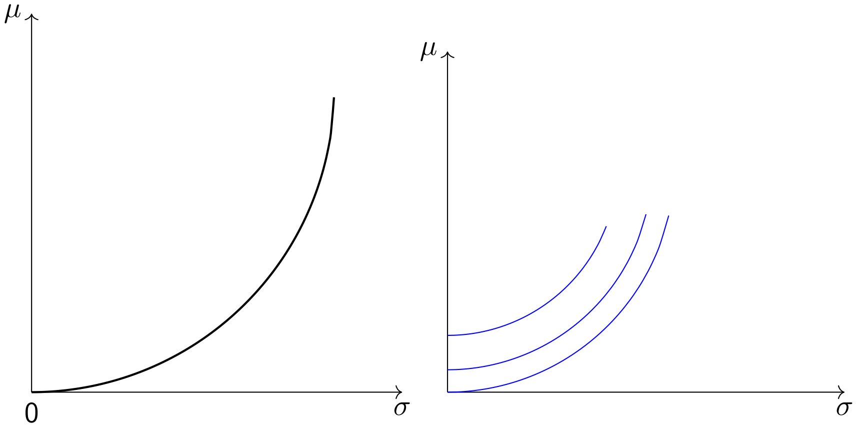
Assets in the same indifference curve

- We can obtain the set of points with the same expected utility (indifference curve):

$$\begin{aligned} \mathrm{E}[U(X)] = k &\iff a\mu - b\left(\sigma^2 + \mu^2\right) = k \\ &\iff \mu^2 - \frac{a}{b}\mu + \left(\sigma^2 + \frac{k}{b}\right) = 0 \\ &\iff \mu = \frac{a}{2b} \pm \sqrt{\left(\frac{a}{2b}\right)^2 - \left(\sigma^2 + \frac{k}{b}\right)} \end{aligned}$$

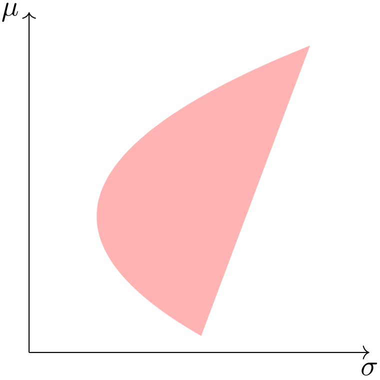
- Since the support of X is contained in $\left[0, \frac{a}{2b}\right]$, it does not make sense the "plus" signal in $\pm$ above, because this would imply $\mu > \frac{a}{2b}$.
- We can plot the above curve (with the minus sign) in the Mean-Variance axes

Indifference curves



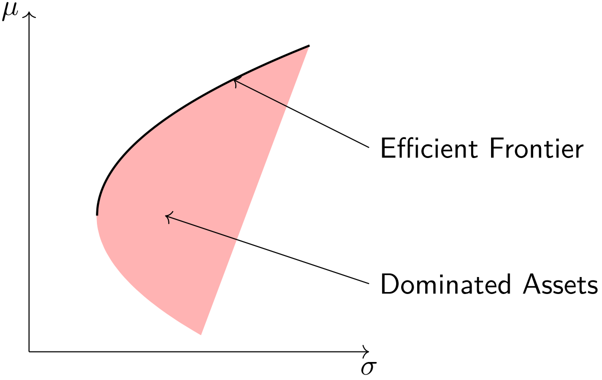
Set of assets in the market

- In any given market, there are a set of assets available, with different means and variances/standard deviations.
- We can compare those assets as shown before



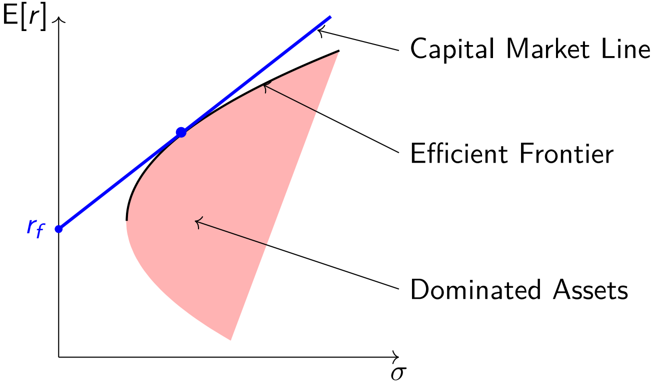
Efficient Frontier

- Some of the assets in the market will dominated (there are others that are strictly better);
- The set of undominated assets form the "Efficient Frontier"



Capital Market Line (CML)

- If there is a risk free asset, we can connect this asset with a point in the Efficient Frontier.
- The line thus formed is called the Capital Market Line.



Capital Asset Pricing Model (CAPM)

Assumptions:

- Markets are frictionless
- Investors care only about their expected mean and variance of their returns over a given period
- Investors have homogeneous beliefs

CAPM Formula: Let M denote the market portfolio. For any asset i,

$$\mathrm{E}[r_i] - r_f = \beta_i \left(\mathrm{E}[r_M] - r_f\right),$$

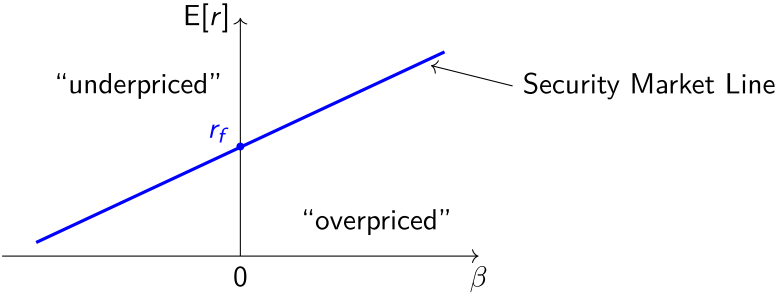
where

$$\beta_i = \frac{cov(r_i, r_M)}{\mathrm{Var}[r_M]} = \frac{\mathrm{E}[r_i r_M] - \mathrm{E}[r_i]\mathrm{E}[r_M]}{\mathrm{Var}[r_M]}$$

CAPM examples

- The CAPM formula leads us to the Security Market Line (SML)

$$E[r_i] - r_f = \beta_i \left(E[r_M] - r_f \right),$$



Structure of assets

- There are n states of the Nature, $S = \{s_1, ..., s_n\}$.
- There are m assets.
- a_{ij} is the payment of asset $i \in \{1, ..., m\}$ if the state of the nature is $j \in S$.
- $A = (a_{ij}) \in \mathbb{R}^{m \times n}$ is the matrix of payoffs.
- $y = (y_1, ..., y_m)$ is the portfolio of assets; y_i is the quantity of asset i that is acquired (bought).
- $p = (p_1, ..., p_m) \in \mathbb{R}^m \setminus \{0\}$ is the vector of prices of assets; p_i is asset i's price.

Definition (Arbitrage)

We say that the above structure does not allow arbitrage if there is no $y \in \mathbb{R}^m$ such that $y \cdot A \geqslant 0$ e $p \cdot y < 0$.

Existence of a risk neutral probability

Definition (Arbitrage)

We say that the above structure does not allow arbitrage if there is no $y \in \mathbb{R}^m$ such that $y \cdot A \geqslant 0$ e $p \cdot y < 0$.

Definition (Risk Neutral Probability)

We say that a probability $\pi : 2^S \to [0,1]$ is a risk neutral probability if there exists a number $\lambda \in \mathbb{R}_{++}$ such that for each asset $i \in \{1, ..., m\}$,

$$p_i = \lambda \mathsf{E}_\pi [a_i] = \lambda \sum_{j=1}^{n} \pi(\{s_j\}) a_{ij}.$$

Existence of a risk neutral probability

Theorem

There is no arbitrage if and only if there exist a risk neutral probability.

Outline

Conclusion

In this course, we have covered the following topics:

1. Basic concepts:
    - Opportunity costs
    - Positive (descriptive) approach
    - Normative (prescriptive) approach
    - Some findings from Behavioral Economics
2. Five steps to Decision Making
3. Decision Under Uncertainty
    - Expected values (means), variance and standard deviation
    - Expected Utility
    - CAPM
    - Arbitrage Pricing

You can learn more about the above issues in the following sources:

- Mas-Colell, Whinston, and Green (1995)
- Duffie (2010)
- Campbell (2017)

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